HOW MOST RECENT GLOBAL GEOPOTENTIAL MODELS FIT THE CROATIAN TERRITORY?

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Abstract. Independent quality control of the national geoid model HRG2000 was performed using 65 control points, obtained through the realization of EUVN and EUVN_DA projects and Croatian fundamental gravity network. Ellipsoidal heights and positions of control points are precisely defined by GNSS measurements while the geodetic heights are obtained by geometric levelling.

Latest geoid model HRG2000 is related to the old vertical system. A comparison analysis indicates that there is a need for new and better national geoid model calculation which has to be connected to new vertical system HVRS71. As preparation and first step, an analysis **of** how global geopotential models fit Croatian territory was made, using eleven recent CHAMP and/or GRACE global geopotential models. The statistic**s** of the differences between GNSS/levelling height anomalies (N_{HVRS71}) and corresponding anomalies obtained from global geopotential models (N_{MODEL}) shows that, without any doubt, EGM2008 model is most suitable at this moment. Also, another interesting fact is that all global models are approximately 93 cm above the new Croatian vertical datum.

Keywords. Croatian geoid model HRG2000, national vertical systems TRIESTE and HVRS71, global geopotential models, EGM2008

1. Introduction

In the Republic of Croatia there are two vertical systems currently in use. The old one, known as TRIESTE, and the new one called HVRS71. By the year 2010 Croatia should change the old vertical system with the new one. The only national geoid model, HRG2000, is related to the old vertical system. This analysis is the preparation for the modelling of the national geoid model that will be connected to the new vertical system. The differences of height anomalies calculated from control GNSS/levelling points and those from global geopotential models are the indicator of which global geopotential model is the best for describing Earth's gravity field on our territory. Global geopotential model is usually used for the definition of long wavelength gravity field structures in geoid modelling.

The differences between height anomalies that were attained from three different sources of data were used in the analysis.

– **The First group** of data represents the height anomalies that were calculated from ground GNSS/levelling data that was made on EUVN, EUVN_DA and FGN (Fundamental Gravimetric Network) points (h_{GNSS}-H_{HVRS71} = ζ_{HVRS71}).
– **The Second group** is height anomalies data that was interpolated from HRG2000 national geoid model ($\zeta_{HRG2000}$), and the
– **Third group** is obtained from eleven global geopotential models (ζ_{MODEL}).

2. Theory background

The property that the height anomaly (ζ) is equal to the distinction between ellipsoidal height (h) and normal (orthometric) height (H^N) was used in the analysis. The main differences between normal and normal-orthometric heights with respect to referenced surfaces and their relationship with height anomaly will be explained in the resumption. Also, a presentation of how global geoid models can be obtained from height anomalies will be made.

2.1 Height systems

Main differences between the orthometric and normal height system are: the avoidance of hypotheses to determine the gravity field inside the topography, the theoretical replacement of the Earth's surface by telluroid and the use of reference ellipsoid with associated gravity field, see Featherstone and Kuhn (2006).

The telluroid is an auxiliary surface obtained by the point-wise projection of points P on the Earth's surface along the straight-line ellipsoidal, normal to points Q that have the same gravity potential value in the normal gravity field U_Q as the original points

P in the Earth's gravity field W_P, $U_Q=W_P$. Thus, the telluroid is not an equipotential surface. The normal gravity field U is an approximation of the real Earth's gravity field W and conceptually generated by masses within a reference ellipsoid. Furthermore, the surface of the reference ellipsoid is a normal equipotential surface that has the same normal gravity potential value as the geoid surface, with respect to the Earth's gravity field, $U_0=W_0$.

The distance, measured along the straight-line ellipsoidal normal, between the Earth's surface (point P) and the telluroid (point Q), is called the height anomaly (ζ), see Figure 1. The distance, measured along the curved normal gravity plumb line, between the reference ellipsoid surface (point Q_0^N) and the projected point Q on the telluroid, is the normal height (H^N). Normal height (H^N) and height anomaly are defined like (ζ), see Torge (1989):

$$H^N = \frac{C}{\bar{\gamma}} = \frac{W_0 - W_P}{\bar{\gamma}} = \frac{W_0 - W_P}{\bar{\gamma}}, \tag{3}$$

$$\zeta = \frac{W_P - U_P}{\gamma_Q} \tag{4}$$

where C is geopotential number in g.p.u.. In other words, normal height is obtained by using the integral-mean value of normal gravity ($\bar{\gamma}$), taken along the normal plumb line between the ellipsoidal surface and the projected point on the telluroid, which is simple to compute and does not use any topographic mass-density data.

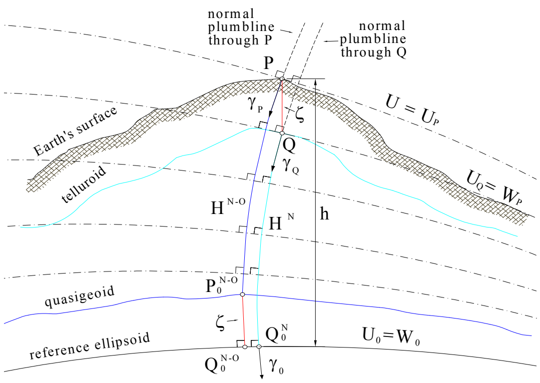

Fig. 1 Normal and normal-orthometric heights

The relationship between ellipsoidal height and normal height, if we neglected curvature of normal gravity plumb line, see Torge (2001), is:

$$h = H^N + \zeta \tag{5}$$

If we apply height anomaly from reference ellipsoidal surface up, we obtain quasigeoid surface. This surface Molodensky called quasigeoid. The guasigeoid height ζ is the straight-line distance reckoned along the ellipsoidal surface, normal from point Q_0^{N-O} on the surface of the ellipsoid to the point P_0^{N-O} on the surface of the quasigeoid and, by definition, have the same length like the height anomaly ζ, see Figure 1.

The quasigeoid is a non-equipotential surface, like telluroid, of the Earth's gravity field and he coincides reasonably closely with the geoid ($N \approx \zeta$) over the oceans but not over the land; up to about 3.4 m in the Himalayas, see Rapp (1997).

Normal orthometric height system is neither an orthometric height system nor a normal height system. Both orthometric and normal height systems require actual gravity observations to be taken along the levelling traverse in order to derive the geopotential numbers (or orthometric or normal corrections). On the other hand, the normal-orthometric system uses only the normal gravity field as an approximation of the Earth's gravity field to derive all related gravity field quantities. Actual geopotential numbers are replaced by the differences in the corresponding normal potential (called normal-geopotential or spheropotential numbers) and actual gravity is replaced by normal gravity.

The geometric interpretation of normal-orthometric height (H^{N-O}) is analogous with that of normal height (H^N), except that the relations are reversed. The height anomaly becomes the separation between the reference ellipsoid (point Q_0^{N-O}) and the quasigeoid (point P_0^{N-O}) measured along the ellipsoidal normal and the normal-orthometric height is the distance between the quasigeoid and the point on the Earth's surface measured along the normal plumb line. The reference surface for normal-orthometric height is the quasigeoid. Normal-orthometric heights can be computed in practice from levelled height differences using the normal-orthometric correction, see Torge (2001).

2.2 Global geopotential models

From global geopotential models one can calculate different quantities (height anomaly) that are connecting actual and normal gravity field, i.e. physical and mathematical surfaces, through disturbing potential.

Disturbing quantities in gravity field are the differences between the actual and the normal gravity field, hence:

$$T_P = W_P - U_P \tag{7}$$

If the actual gravity field is associated with the normal gravity field with the following conditions:

$$U_Q = W_P, \quad U_0 = W_0 \tag{8}$$

and if the mass of the level ellipsoid is equal to that of the Earth, then the spherical-harmonic expansion of gravity disturbing potential T defined at the point P yield to, see Hećimović and Bašić (2002):

$$T = \frac{GM}{r}\sum_{l=2}^{l_{max}}\left(\frac{a}{r}\right)^l \sum_{m=0}^{l}\left(\Delta\overline{C}_{lm}\cos m\lambda + \Delta\overline{S}_{lm}\sin m\lambda\right)\overline{P}_{lm}(\cos\Theta) \tag{9}$$

where GM is geocentric gravitational constant referring to the total mass (earth's body plus atmosphere), (r, λ, Θ) – spherical coordinates, l(n), m – the degree and order of the expansion of global geopotential model, a – semi major axis of the ellipsoidal earth model, $\Delta\overline{C}_{l,m}$ and $\Delta\overline{S}_{l,m}$ - the differences between the fully normalised spherical harmonic coefficient of the actual and normal gravity field, $\overline{P}_{lm}(\cos\Theta)$ - fully normalized Legendre polynomials, l_{max} – maximal expansion of global geopotential model.

From the disturbing potential, different quantities can be calculated: height anomaly (ζ), gravity disturbance (δg), gravity anomaly (Δg), component of vertical deflection (ξ, μ).

The formula that relates the height anomaly with the disturbing potential is well known Bruns formula, see Hofmann-Wellenhof and Moritz (2006):

$$\zeta = \left(\frac{T}{\gamma}\right)_Q \tag{10}$$

where γ_Q is ellipsoidal normal gravity, see formula (4).

A spherical harmonic expansion for the height anomaly is obtained when the equation (9) is divided by the normal gravity γ_Q.

3. Used data

3.1. Set of control points

65 EUVN, EUVN_DA and FGN control points were used in the analysis of the differences between heights anomalies.

EUVN (EUropean Vertical Network) project in Croatia was realized in 1997. Eight EUVN points were defined, see Marjanović and Rašić (1998). The first discrepancy analysis of geoid heights points up the existence of some suspicious points. In Croatia, there were two problematic points (HR05 Split and HR01 Bakar), see Kenyeres et al. (2002).

The problem with these, and few other points, was solved through EUVN Densification Action (EUVN_DA) project in 2005, after Croatian Geodetic Institute took the coordination of the project. New densification points were selected according to the points from 1997, the shape and the topography of Croatian territory. The number of EUVN sites has expanded from 8 to 20. With the realization of the project, GNSS/levelling/ gravimetric points were defined. EUVN heights are determined in both new and old Croatian heights systems.

The set of control points included, beside the twenty EUVN and EUVN_DA points, 45 points of Croatian Fundamental Gravity Network (FGN). Ellipsoidal heights and positions are precisely defined by GNSS measurements while the geodetic heights are obtained by geometric levelling. The heights of all points, EUVN and FGN, are expressed in old (TRIESTE) and new (HVRS71) national vertical system.

The greater value of this set of points is that they all expressed in both height systems (normal-orthometric heights) and that all points have GNSS/levelling/gravimetric data.

3.2. HVRS71 and TRIESTE height datum

In the Republic of Croatia there are two vertical systems currently in use. The old one known as TRIESTE and the new one named HVRS71.

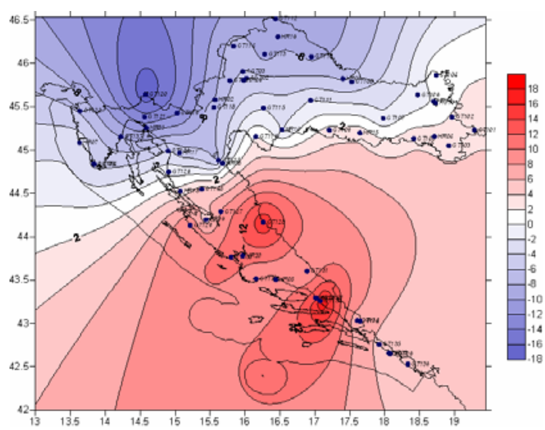

Fig. 2 Distinction between HVRS71 (new) and TRIESTE (old) vertical system (H_{HVRS}-$H_{TRIESTE}$)

Old vertical system is defined for the epoch 1875.0 by the averaging of one-year tide gauge measurements in Trieste. New vertical datum is based on zero-surface reference determined by the averaging of sea level data observed over 18.6 year period at five tide gauges along the Croatian (four) and Slovenian (one) part of the Adriatic coast and defined at the epoch 1971.5. The extension of vertical datum to the continental part of Croatia has been realized through the levelling of high order and normal orthometric heights. On Figure 2. one can see the differences in height (H_{HVRS}-$H_{TRIESTE}$) between the two systems in discrete points when the shift of 22.7 cm is removed, in other words, the HVRS71 datum is for 22,7 cm higher then TRIESTE.

3.3 HRG2000 national geoid model

The Republic of Croatia has an unpleasant shape for geoid modelling. Furthermore, the Adriatic coast is extremely demanding for geoid modelling as very high mountain massif is rising fast from sea, due to big geoid gradients are present. In Croatia, HRG2000 (HRvatski Geoid 2000) national geoid model is currently in use, and it is fitted to the old Croatian height system TRIESTE.

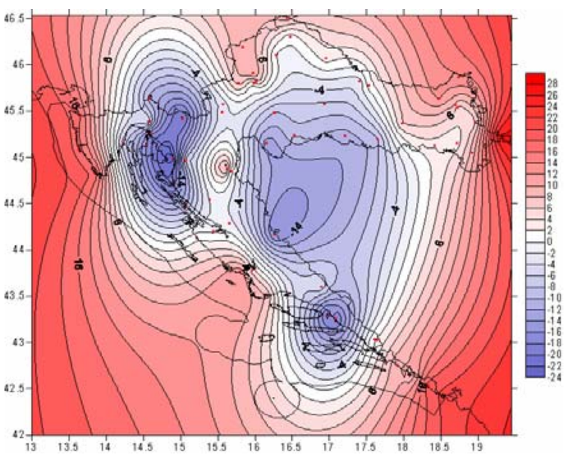

Fig. 3. Difference between height anomalies ($\zeta_{TRIESTE}$-$\zeta_{HRG2000}$)

HRG2000 geoid model was developed in the Department for Geomatics at the Faculty of Geodesy, University of Zagreb, in 2000, see Bašić (2001). State Geodetic Administration proclaimed HRG2000 as the official geoid surface of the Republic of Croatia. Geoid model was calculated by least square collocation method. Following input data were used: global geopotential model EGM96, free-air gravity anomalies, geoid undulations from GNSS/levelling data and high-resolution digital terrain models. For interpolation height anomalies (input data - geographic latitude and longitude), a computer program was created: IHRG2000. In IHRG2000 program there are two possibilities of interpolation: bilinear and spline, see Bašić and Šljivarić (2003) and Bašić and Hećimović (2006). In this analysis bilinear interpolation was used.

Figure 3. shows height anomaly differences ($\zeta_{TRIESTE}$-$\zeta_{HRG2000}$) in cm, at discrete points, between the old national vertical system TRST and the national geoid model HRG2000. Statistical values are represented in Table 1.

Table 1. Statistical values of height anomalies differences in discrete points of control set

Height anomald ifferen-ces	AVER-AGE [cm]	ST. DEV [cm]	MIN [cm]	MAX [cm]	RAN-GE [cm]
$\zeta_{TRIESTE}$-$\zeta_{HRG2000}$	0.47	11.0	-22.8	26.0	48.8

3.4 Global geopotential models

Height anomalies data that were obtained from web pages of International Centre for Global Earth Models (ICGEM) (URL1) were used in the analysis. The setting was: refsys – GRS80, functional – height anomaly, tide system – use unmodified model, gridstep – 0.01°, grid boundaries – 13° - 19.45° E, 42° – 46.54° N. After the data was downloaded, the Scripter of Surfer 8 software was used to perform bilinear interpolation of Z (ζ) value (grid1.Interpolate(x,y)).

Table 2. Global geopotential models that are presented in this article

MODEL	Max. degree	Year	Data
EGM2008	2190x2159	2008	S(Grace),G,A
EGM2008 (360)	360x360	2008	S(Grace),G,A
EGM96	360x360	1996	EGM96S,G,A
EIGEN-CGO1C	360x360	2004	S(Champ, Grace),G,A
EIGEN-GL04C	360x360	2006	S(Grace, Lageos),G,A

*Abbreviations in Table explain source of the data; S – satellite tracking data, G – gravity data, A – altimetry data.

The results of only five models, from eleven analysed, are carried out in this article, see Table 2. These were chosen since they have better statistical values of height anomaly differences (ζ_{HVRS71}-ζ_{MODEL}) from the six others global models. The following models were also included in the analysis: EIGEN-CG03C, AIUMB-CHAMP01S, ITG-GRACE02S, GGM02C, EIGEN-CHAMP03S and ITG-GRACE03.

In winter 2008, National Geospatial-Intelligence Agency (NGA), EGM Development Team, has publicly released the new global geopotential model, EGM2008. EGM2008 model is complete to the degree and order 2159, and contains additional spherical harmonic coefficients extending to the degree 2190 and the order 2159. Taking into consideration that the "old" models have been expanded to the degree and order 360, the results are not so surprising. EGM2008 incorporates improved 5x5´ gravity anomalies and latest GRACE based satellite solution (ITG-GRACE03S). EGM2008 also includes improved altimetry-derived gravity anomalies, estimated using PGM2007B model and its implied Dynamic Ocean Topography (DOT) model as reference, see Pavlis et al. (2008). Also available is the solution of EGM2008 model that is expanded to the degree and order 360 (EGM2008(360)) (URL 2).

4. RESULTS

Height anomalies that were attained from three different sources of data were used in the analysis.

Table 3. Statistical values of height anomalies from three sources of data

Height anomaly	AVE-RAGE [m]	ST. DEV. [m]	MIN. [m]	MAX. [m]	RAN-GE [m]
National vertical systems					
$\zeta_{TRIESTE}$	43.977	1.467	40.112	46.823	6.711
ζ_{HVRS71}	44.204	1.415	40.380	46.877	6.497
National geoid model HRG2000					
$\zeta_{HRG2000}$	43.972	1.493	39.920	46.930	7.010
Global geopotential models					
$\zeta_{EGM2008}$	45.103	1.369	41.483	47.603	6.120
$\zeta_{EGM2008 (360)}$	45.095	1.389	41.275	47.162	5.888
ζ_{EGM96}	45.358	1.423	40.571	47.188	6.617
ζ_{CGO1C}	45.058	1.437	40.950	47.150	6.201
ζ_{GL04C}	45.078	1.453	40.783	47.011	6.228

The first group of data in Table 3. represents the height anomalies that were calculated from ground GNSS/levelling data, made on EUVN, EUVN_DA and FGN (Fundamental Gravimetric Network) points (h_{GNSS}-H_{HVRS71}=ζ_{HVRS71}, h_{GNSS}-$H_{TRIESTE}$=$\zeta_{TRIESTE}$). The second and the third group of data was acquired by interpolation from HRG2000 national geoid model ($\zeta_{HRG2000}$) and global geopotential models (ζ_{MODEL}).

Average (m) in Table 3. is the arithmetic mean of height anomalies, calculated for discrete points, that refers to the height of reference surfaces above the reference ellipsoid GRS80. Reference surfaces are: TRIESTE and HVRS71 vertical system, geoid surface HRG2000 and five global geopotential surfaces.

Table 4. presents the statistical values of height anomaly differences (ζ_{HVRS71}-ζ_{MODEL}) that were obtained by the detraction of HVRS71 height anomalies from height anomalies interpolated from global geopotential models at discrete points. Referent values were height anomalies calculated from GPS/levelling data and expressed in new Croatian vertical system HVRS71 (ζ_{HVRS71}=h_{GNSS}-H_{HVRS71}). The vertical system HVRS71 is chosen as a referent because the purpose of this analysis was to contrive global geopotential model that will be used like an input model in the calculation for the new Croatian local geoid model, that will refer to HVRS71 datum.

Table 4. Statistical values calculated for height anomaly differences between height anomalies that refer to Croatian datum HVRS71 (ζ_{HVRS71}) and those refer to global geopotential models (ζ_{MODEL}).

HVRS71 datum (ζ_{HVRS71}-ζ_{MODEL})					
MODEL	AVERAGE [cm]	ST.DEV. [cm]	MIN [cm]	MAX [cm]	RANGE [cm]
HRG2000*	23.2	13.0	-10.0	52.4	62.4
EGM2008	-89.9	8.8	-113.5	-66.0	47.5
EGM2008 (360)	-89.1	21.1	-146.8	-28.6	118.2
EGM96	-115.4	29.7	-184.7	-19.1	165.7
EIGEN-CGO1C	-85.4	22.7	-137.3	-27.4	109.9
EIGEN-GL04C	-87.4	23.8	-143.6	-13.5	130.2

* National model HRG2000 is referred to TRIESTE datum, existence of shift between two datum's of 23 cm in discrete points.

From calculated statistical values (standard deviation) it is obvious that the global geopotential model EGM2008 fits best to the Earth's gravity field on the territory of Croatia. The EGM2008 model has approximately three times better statistical values then other global models, see Table 4. In Figure 4. it can be seen that maximum and minimum differences (-23,9 cm and 23,5 cm), when the shift of 89,9 cm is removed, pop up at points with greater normal-orthometric heights (mountains), in area near the national border and at points of great disproportion in height between sea surface and land (Dinaride mountain massif). GNSS/levelling data of the critical points need to be re-examined at the field and the sources of errors established. EGM2008 is 89,9 cm above HVRS71 datum and 112,6 cm above TRIESTE datum.

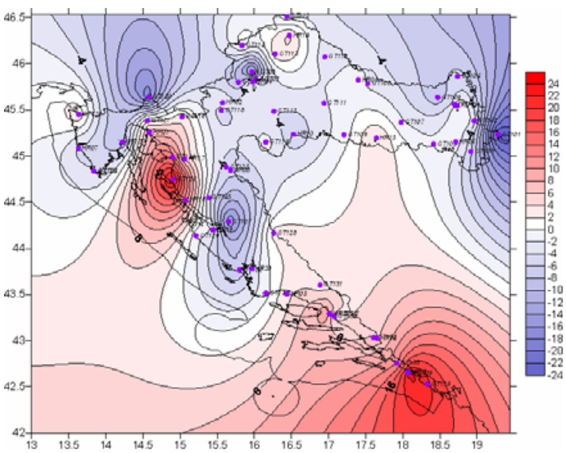

Fig. 4. Differences ζ_{HVRS71}-$\zeta_{EGM2008}$ when the shift of 89,9 cm is removed

Global model EGM2008 (360), that is expanded to the degree and order 360, also has better statistical indicators than other models with the same expansion, see Table 4. Before the EGM2008 model was published, models EIGEN-CG01C and EIGEN-GL04C were the models with the best statistical values for Croatian territory. If all global models are taken into account, HVRS71 datum is approximately 93,2 cm below the referent surfaces of global geopotential models.

5. CONCLUSIONS

The purpose of the analysis of eleven global geopotential models was to find the most suitable model regarding Earth's gravity field along Croatian territory. In this paper five models that give the best statistical results were presented. Without any doubt, the newest global model, EGM2008, showed the best results. With EGM2008 model the new era in geoid modelling started. As was already pointed out, the EGM2008 has three times better statistical indicators than other "old" global models, expanded "only" to the degree and order 360. It will be interesting to see the results of the modelling of new Croatian geoid model with the presumption that EGM2008 will be used for the definition of long-wavelength gravity structures in calculation procedure.

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