

Extension of the Croatian Fundamental gravity network to the north Adriatic islands

M. Repanić, B. Barišić, I. Grgić, M. Liker, M. Lučić, T. Bašić
Croatian geodetic institute, Savska c. 41/XVI, 10144 Zagreb, p.p.19, Croatia.

Abstract. In September and November 2007 Croatian Fundamental gravity network (FGN) has been extended to north Adriatic islands by relative gravimetry. The paper describes FGN extension measurements, applied adjustment models and analyses of results with critical remarks.

In order to ensure homogeneity of the network, the extension has been designed in accordance with initial FGN, apart from station separation and mean of transportation.

The extension has been adjusted according to the least squares principle, first as independent free network, subsequently as biased network with stations from present FGN fixed. Two adjustment models have been applied for both datum definitions. For one model of adjustment daily drift corrections have been determined prior to adjustment, while in second introduced as parameters in functional model. For both models of adjustment, stochastic model is based on variances of gravity readings but involves total mathematical correlation and estimated uncertainty of daily drift corrections (if determined prior to adjustment). Although drift values differs significantly, estimated gravity values from two models show no significant difference (up to $0.008 \mu\text{ms}^{-2}$). Standard deviations of estimated gravity values for model 1 and 2 do not exceed $0.08 \mu\text{ms}^{-2}$ and $0.09 \mu\text{ms}^{-2}$, respectively.

All computations have been preformed automatically by several programming functions developed in Matlab.

Since within next 2 years FGN extension to all significant Adriatic islands is planned, described extension procedure is only a first phase that shall provide valuable experience in forthcoming projects.

Keywords. Gravity network, relative gravimetry, adjustment models.

1 Introduction

Before extension, Croatian Fundamental gravity network (hereafter FGN) comprised 42 stations, from which 6 station are zero order stations, with gravity values determined by absolute measurements in period form 1996 to 2000, and 36 first order stations (Barišić et al 2008) (Fig. 1.1). Since absolute station AGT05 have been destroyed, it has been replaced by its eccenter AGT05E, with gravity value determined by relative measurements.

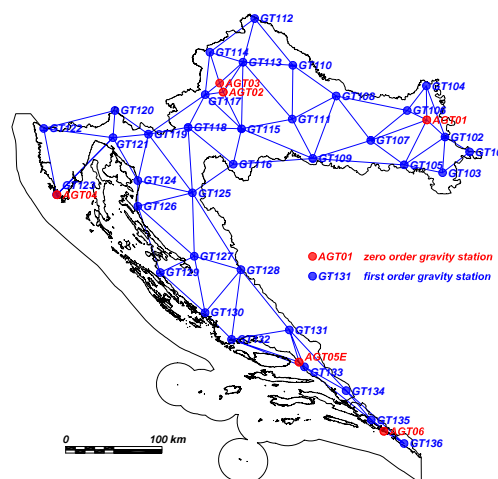


Fig. 1.1: Fundamental gravity network before extension

In September and November 2007 FGN has been extended to north Adriatic islands, as common project of Croatian geodetic institute and Faculty of Geodesy, University of Zagreb. The extension (Fig. 1.2) comprises 16 stations in total: 7 old and 9 new stations, from which 7 on islands and 2 on mainland (for the purpose of network densification). In order to ensure homogeneity of the network, the extension has been designed in accordance with initial FGN, regarding number of relative gravity ties of each station to adjacent stations, measurements methods (star and profile) and number of observations of each tie.

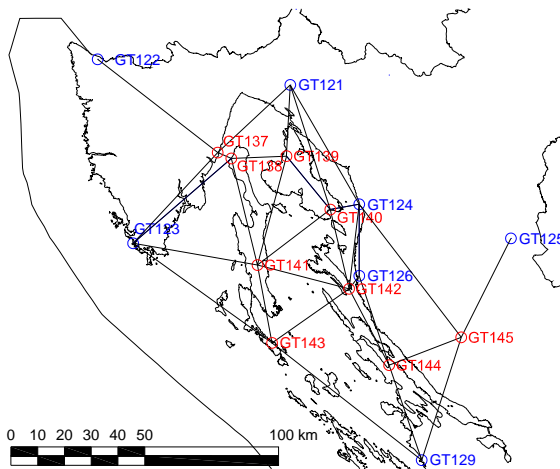


Fig. 1.2: FGN extension to north Adriatic islands

2 Relative gravity measurements

As in FGN, the network is designed as a system of triangles, with each gravity tie measured twice (in different directions) with same 3 gravimeters simultaneously. Namely, used instruments are two Scintrex Autograv CG-3M, owned by Croatian geodetic institute, and one CG-5, owned by Faculty of Geodesy. Though, extension significantly differs from initial FGN in station separation and mean of transportation. Specifically, while for the present FGN gravimeters have been transported only by car, for extension car, ferry boat, catamaran and speedboat have been used. Extension consists of 31 gravity ties that have been measured in two directions with three instruments that make 186 relative gravity measurements. Measurements have been carried out in 14 days. However, because of problems with CG-5 batteries, pause of more than a month has been made before last 4 days.

Besides regularly performed control of gravimeters' operation (Barišić and Repanić 2007), before campaign, all three gravimeters have been calibrated on provisory calibration range that comprises only two absolute gravity stations AGT02 and AGT03. Experience of several years of relative gravimeters calibration on mentioned range provided considerably large variations of calibration parameters. For now there is too few data to make conclusion whether the variations are caused by measurements of poor quality, or by variation of mass distribution in vicinity of AGT02, which is surrounded by lakes. However, it is necessary to determine calibration correction function in the course of network adjustment.

3 Data processing

To eliminate systematic influence, instrumental corrections and reductions for temporal gravity changes and gravimeter displacement have been applied to gravity readings.

Scintrex Autograv CG-3M and CG-5 are modern gravimeters with automatic electronic and software control of the measurement process. Hence, instrumental corrections (i.e. stationary drift correction, tilt correction and temperature compensation) are automatically applied by the instruments. In addition, the gravimeters calculate gravimetric tidal reduction according to Longman formula and (if selected) applies it to gravity reading in real time. Data on basis on which corrections are calculated (temperature and tilt values, etc.) is stored in the instruments' memory and displayed on the printout, together with tide reductions, gravity readings and other relevant data. Specific formulas for instrumental corrections can be found in Scintrex 1998, 2006.

In addition to instrumental correction and tide reductions, barometric and height reduction were applied to measurement prior to adjustment. Reductions were calculated according to formulas given in Torge 1989. Height reduction has been calculated on the basis of normal vertical gradient. To reduce influences of difference between normal and real gradient value on measurements precision, instruments height have been held constant by fixing one of three foot screws. In addition, instruments have been placed always on same marked place at each station.

For the purpose of control of instrumental influences and to increase precision at least five readings were taken at each station. Mean reading value, with weights calculated on the basis of standard deviations, has been calculated for each station occupation. In addition, remaining reductions have been applied to each mean reading. For the purpose of determination of calibration coefficient correction, raw gravity reading, without applied corrections and reductions, is needed. Therefore, total value of applied corrections and reductions (apart from stationary drift correction) has been calculated for each mean reading. The calculations are made automatically by programming function developed in Matlab.

4 Applied adjustment models

The extension has been adjusted according to well known Gauss-Markov model and least square principle, first as independent free network, and subsequently as biased network with 7 stations from initial FGN fixed. Such biased adjustment is not an optimal choice, since, in this way, uncertainties of old fixed stations are ignored and passed to new stations. It would be more appropriate to treat all first order stations in same way. Thus, to adjust whole network, i.e. initial FGN and extension, at once, with only zero order stations fixed. However, at this moment it is the most economic choice, since further extensions to central and south Adriatic islands are planed in next few years, and, otherwise, every new extension phase would mean new gravity values for old stations.

Two adjustment models have been applied. In both adjustment models, functional model takes in differences of successive corrected reading as functions of unknown parameters, i.e. gravity values, linear calibration parameters and, optionally, linear daily drift corrections. Main difference between two models is that, in model 1, daily drift corrections have been determined prior to adjustment, from repeated observations of identical stations in one measuring day, while in the model 2, daily drift corrections have been determined as unknown parameters in the course of adjustment. In both adjustment models, stochastic model is based on variances of mean gravity readings, but involves total mathematical correlation and estimated uncertainty of daily drift corrections (if determined prior to adjustment). Both functional models are based on the model given in Torge 1989.

4.1 Model 1

Linear daily drift corrections have been determined from independent adjustments for each measuring day and gravimeter. Measurement equitation of such daily adjustment is:

$$z_T^c(t_i) + v_i = z_T^c(t_0) + d_1^{d,gr}(t_i - t_0), \quad (1)$$

where $z_T^c(t_i)$ is corrected gravity reading at time of observation t_i , $z_T^c(t_0)$ corrected gravity reading at reference time t_0 , and $d_1^{d,gr}$ linear drift parameter for day d and gravimeter gr . Reading weights have been defined as inversely proportional to readings' variances.

The values that are introduced in adjustment of the gravity network as measurements have been

computed as difference of subsequent corrected gravity readings, previously reduced for daily drift correction:

$$L_k = z_j^c - d_1^{d,gr}(t_j - t_0) - (z_i^c - d_1^{d,gr}(t_i - t_0)). \quad (2)$$

Thus, measurements are derived from gravity readings, according to known functions (2). Weight matrix can also be computed, according to weight propagation law (Feil 1989, 1990). Moreover, since the measurements of same measuring day and same gravimeter are derived from same group of readings, they are correlated. Therefore, the weight matrix of computed measurements shall consist of symmetric block matrices, each corresponding to one measuring day and gravimeter:

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}^{1,1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^{2,1} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{P}^{14,3} \end{bmatrix}, \quad (2)$$

where block matrices for each measuring day (d) and gravimeter (gr) are computed from:

$$(\mathbf{P}^{d,gr})^{-1} = \mathbf{A}_\Delta \cdot (\mathbf{P}_{z-d}^{d,gr})^{-1} \cdot \mathbf{A}_\Delta^T, \quad (3)$$

where $\mathbf{P}_{z-d}^{d,gr}$ is weight matrix of corrected gravity readings of day d and gravimeter gr , reduced for daily drift correction:

$$(\mathbf{P}_{z-d}^{d,gr})^{-1} = (\mathbf{P}_z^{d,gr})^{-1} + \mathbf{a} \cdot \mathbf{q}_{d,gr} \cdot \mathbf{a}^T, \quad (4)$$

and:

$$\mathbf{A}_\Delta = \begin{bmatrix} -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix}, \quad (5)$$

$$\mathbf{a}^T = [(t_0 - t_1) \quad (t_0 - t_2) \quad \dots \quad (t_0 - t_n)]. \quad (6)$$

Matrix $\mathbf{P}_z^{d,gr}$ in (4) is a diagonal weight matrix of independent gravity readings, with weights defined as inversely proportional to readings' variances.

The functional model is based on measurement equitation:

$$L_k + v_k = g^B - g^A - y_i^{gr}(z_j - z_i), \quad (7)$$

where v_k is measurement residual, g^T gravity value at station T , y_i^{gr} , correction of linear calibration parameter for gravimeter gr and z_i gravity reading without corrections and reduction applied.

4.2 Model 2

Second model is much simpler, but involves considerably larger number of unknown parameters. Its functional model is based on measurement equitation:

$$\begin{aligned} L_k + v_k &= \Delta z_{i,j}^c + v_k = \\ &= g^B - g^A - y_i^{gr} (z_j - z_i) + d_i^{d,gr} (t_j - t_i). \end{aligned} \quad (8)$$

Stochastic model is derived analogously as in previous adjustment model. The weight matrix of computed measurements (2) consists of symmetric block matrices, each corresponding to one measuring day and gravimeter. However, those are computed according to:

$$(\mathbf{P}^{d,gr})^{-1} = \mathbf{A}_\Delta \cdot (\mathbf{P}_z^{d,gr})^{-1} \cdot \mathbf{A}_\Delta^T, \quad (9)$$

where $\mathbf{P}_z^{d,gr}$ is a diagonal weight matrix of independent gravity readings, with weights defined as inversely proportional to readings' variances, and \mathbf{A}_Δ is defined as before (5).

5 Adjustment results

As described above, the extension has been adjusted according to two adjustment models. For both models, first free network adjustment has been carried out, and then biased adjustment, that should provide gravity values of new stations for first phase of extension. In addition to adjustment of all three gravimeters' data together, for both models of adjustment and for both datum definitions, separate adjustment for each gravimeter data independent have been performed, to provide valuable control of systematic influences that may not be included in adjustment model. Thus, there have been 16 adjustment carried out in total. All computations have been performed automatically in Matlab. For that purpose, several programming functions have been designed.

The results of adjustment have been compared and evaluated on the basis of several criteria:

- standard deviations of estimated gravity values,
- global accuracy criteria based of variance-covariance matrix of estimated gravity values (sub-matrix of whole variance-covariance matrix),
- standard deviations of other unknown parameters (i.e. corrections of linear calibration parameters and linear daily drift parameters),
- standard deviations of measurements, and

- differences of gravity values from separate adjustments for each gravimeter data.

The comparison showed that separate adjustment of CG-5 data provided somewhat greater standard deviations of measurements and unknown parameters, as well as global accuracy criteria, than for separate adjustment of both CG-3M gravimeters. In addition, difference of gravity values from separate adjustment of CG-5 data and two CG-3Ms data is larger than difference between values from adjustments of two CG-3Ms.

For that reason, detection of outliers has been performed by Tau-test, according to Kavouras 1982 and Kuang 1992, also as automatic process by Matlab programming function. Because of specificity of relative gravimetry data processing and network adjustment, standard outlier detection procedure (i.e. iterative process of measurement elimination, on the basis of maximum test statistic and examination of related redundancy number, followed by readjustment) hasn't been performed. Explicitly, all data from same measurement day and gravimeter are correlated. Moreover, from measurement market as outlier, it is impossible to conclude which of two related gravity readings contains gross error. Even if it could be located, by eliminating an erroneous reading or two, it could be impossible to use the rest of the readings for related day and gravimeter. For that reason, Tau-test is performed on all 16 adjustments, and its results are compared and evaluated. It primarily indicated problems with CG-5's measurements. Interesting fact is that all outliers of CG-5 measurements that are proven to be caused by gross errors in corresponding measurements (by examination of redundancy numbers according to Kuang 1992) are from the 4th and 7th measuring day (September 22nd and 26th 2007). Furthermore, both days catamaran was used as mean of transportation. The rest of the measurements that are marked as outliers are not marked so consistent through different adjustments, and do not exceed critical value by such large value as CG-5's measurements of day 4 and 7. Results could indicate a problem of Scintrex CG-5 with transportation by catamaran. However, there is not enough data to make such a conclusion, especially given that construction and principles of operation of Scintrex CG-3M and CG-5 is practically the same (Scintrex 1998, 2006). Still, evaluation of measurements based on accuracy criteria and Tau-test undisputedly bring to a decision to exclude all

data of CG-5's day 4 and 7 from network adjustment.

After, eliminating disputable data, again all 16 adjustments have been carried out. Results showed significant accuracy improvement. All accuracy criteria from separate adjustment of CG-5's data have been considerably improved, as well as, criteria from common adjustment of all three gravimeters' data. In addition, differences of gravity values from separate adjustments don't significantly differ neither for one gravimeter.

Although adjusted drift parameters differ significantly, adjusted gravity values from two applied models show no significant difference (up to $0.008 \mu\text{ms}^{-2}$ for common adjustment of all three gravimeters' data), that implies correctly defined stochastic models. Standard deviations of gravity values from common adjustment according to model 1 are smaller, for averagely $0.01 \mu\text{ms}^{-2}$, than from model 2 (up to $0.08 \mu\text{ms}^{-2}$ and $0.09 \mu\text{ms}^{-2}$, respectively). On the other hand, gravity values from separate adjustments for each gravimeter data alone better agree for model 2 than for model 1. Altogether, it is hard to put one model in precedence over another. While model 1 gives better results as concerned local and global accuracy criteria, model 2 provides better agreement of separate adjustments of each gravimeter's data alone, that could indicate that systematic influences are better described by model 2. Further more, although stochastic model of model 1 incorporates total mathematical correlation and takes in account uncertainties of daily drift corrections applied to measurements prior to adjustment, it could be possible that uncertainty estimation provided by model 1 is too optimistic. However, since differences of gravity values from two models are far below accomplished and declared accuracy, and since differences in accuracy criteria are practically insignificant, which model to choose is only the matter of preference of one processing procedure over another.

6 Conclusion

Within next 2 years FGN extension to all significant Adriatic islands is planned in two more phases. One should keep in mind that present results of first extension are only temporarily and that, to ensure equal treatment of all first order stations, after the measurements of least phase are completed, the whole network should be adjusted at once. In addition, densification of Croatian gravity

network with second order network already started in first quarter of this year with measurements in the Istria region. Thus, described extension procedure and adjustment models analysis is only a first step that shall provide valuable experience in forthcoming projects.

It is worth to stress the advantage achieved by the use of modern processing methods. Use of software tools that enable both computing and programming methods easy to use (such as Matlab), unshackle geodesists beyond the limits of commercial processing software and enable them to put to the test different, and perhaps new, methods.

References

- Barišić B, Repanić M (2007). Report on regularly control of relative gravimeters HGI-1 and HGI-2 for time period from 1st of January to 30th of June 2007 (in Croatian), Croatian geodetic institute, Zagreb.
- Barišić B, Repanić M, Grgić I, Bašić T, Liker M, Lučić M, Markovinović D (2008). Gravity measurements on the territory of the Republic of Croatia – past, current and future gravity networks. In: *Proc. Of International symposium on Terrestrial gravimetry: Static and mobile measurements*, Elektropribor, Saint Petersburg, pp 197-202.
- Feil L (1989). *Theory of errors and least squares adjustment – part one* (in Croatian), Faculty of Geodesy, University of Zagreb.
- Feil L (1990). *Theory of errors and least squares adjustment – part two* (in Croatian), Faculty of Geodesy, University of Zagreb.
- Kavouras M (1982). On the Detection of Outliers and the Determination of Reliability in Geodetic Networks, *University of New Brunswick Technical Report no 87*, Fredericton.
- Kuang S (1992). *Analytical Models of Optimal First Order Design of Geodetic Networks*, Fredericton.
- Scintrex (1998). *CG-3/3M Gravity Meter User's Guide*, Scintrex Limited, Canada.
- Scintrex (2006). *CG-5 Scintrex Autograv System Operational Manual*, revision 2, Scintrex Limited, Canada.
- Torge W (1989). *Gravimetry*, de Gruyter, Berlin- New York.